A. Find the limit at $t = 0$ of this curve in $\mathbb{R}^3$:

$$\vec{x}(t) = \left(e^{-3t}, \frac{t^2}{\sin^2 t}, \cos 2t \right).$$

B. For the curve

$$\vec{x}(t) = \left(t^2 + 1, 4\sqrt{t}, e^{t^2 - t} \right)$$

in $\mathbb{R}^3$, first compute $\vec{x}\,'(t)$, then find a parametrization of the tangent line at the point $(2, 4, 1)$.

C. For the curve

$$\vec{x}(t) = \left(\frac{1}{t+1}, \frac{1}{t^2+1}, \frac{t}{t^2+1} \right)$$

in $\mathbb{R}^3$, compute $\int_0^1 \vec{x}(t)\, dt$. (Hint: The antiderivative of $(t^2+1)^{-1}$ is $\arctan t + C$.)

D. Suppose that $\vec{x}$ is a curve in $\mathbb{R}^n$. Compute

$$\frac{d}{dt} \left| \vec{x}(t) \right|.$$

What must you assume about $\vec{x}$, for your answer to make sense? I mean, is your answer defined everywhere? Now restrict to $n = 2$. Does it help to convert your answer to polar coordinates?

E. For curves $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^3$, prove the product rule for the cross product:

$$(\vec{u} \times \vec{v})' = \vec{u}\,' \times \vec{v} + \vec{u} \times \vec{v}\,'.$$

(This is an assigned homework problem.)