There are four problems, but some of them have multiple parts. Sometimes you might find that a problem is hard to decipher, but that it isn't very difficult, once you figure out what it's asking. That's how life is sometimes. :)

Here is a fact, which I asserted in class without proof: If U is an $n \times n$ matrix such that $\|U\vec{v}\| = \|\vec{v}\|$ for all n-dimensional vectors $\vec{v}$, then U is orthogonal. In this first problem, your job is to verify this fact for the 2×2 case. I recommend that you follow these four steps.

A. A matrix A is said to be *symmetric* if $A^\top = A$. Show that $U^\top U$ is symmetric. Therefore the off-diagonal entries of $U^\top U$ must be equal.

B. Apply the assumption that $\|U\vec{v}\| = \|\vec{v}\|$ to the particular vector $\vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. What does it tell you about the entries of $U^\top U$?

C. Now apply the assumption to $\vec{v} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

D. Now apply it to $\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Conclude that $U^\top U = I$.

Second, do Discussion Problem 10 parts C and D, which are about a quantum state collapsing.

Third, prove that if U and V are both $n \times n$ and orthogonal, then UV is also. (Don't let the word "prove" scare you. This is a short algebraic calculation.)

Fourth, do Discussion Problem 12A, which is about Rodrigues's rotation matrix. I am not asking you to hand in parts B and C, although you are welcome to do them, if you want strenuous practice.