There are three problems, but each consists of multiple parts.

For the first problem, let

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ 1 & 1 \\ 0 & 2 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

A. For which number d is the column space of A a subset of d-dimensional space?

B. How would you describe the column space: a zero-dimensional space (that is, a point), or a one-dimensional space (a line), or a two-dimensional space (a plane), etc.?

C. Compute the projection P onto the column space of A, as a matrix of the appropriate dimensions.

D. What is P^2, meaning PP? Why does the answer make sense, based on what P means? While you're at it, tell me what P^k is, for all integers $k \geq 1$.

D. Is the vector $\vec{v}$ above in the column space of A? (Hint: Use P.)

E. For that same $\vec{v}$, let $\vec{w} = \vec{v} - P\vec{v}$. So $\vec{v} = P\vec{v} + \vec{w}$. In English, describe how the two parts of $\vec{v}$ — by which I mean $P\vec{v}$ and $\vec{w}$ — relate to the column space of A. Also, finish this calculation:

$$P\vec{v} = P(P\vec{v} + \vec{w}) = PP\vec{v} + P\vec{w} = \cdots .$$

For the second problem, let A be an $m \times 1$ matrix — in other words, a column vector $\vec{v}$.

A. Compute the projection P onto the column space of A. State your answer in terms of $\vec{v}$.

B. This operation should match an operation that we discussed in class a little earlier — near the end of Day 7 and the beginning of Day 8, I think. Show that the two operations are equal, by showing that they have the same effect on any given vector $\vec{w}$.

For the third problem, consider these three points in the x-y-plane:

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

We wish to find numbers m, b such that the line $y = mx + b$ passes through the three points.

A. Express this problem as a system of three linear equations in two unknowns. Write it in matrix form.

B. There is no solution, but find the least-squares solution, following the outline given in class.