

First, please do Section 2.8 #6. (The word “generated” is a synonym for “spanned”.) Also do #7, 9, 17, 18.

Next, do Section 2.9#5, 6. (The problem is using fancy language to ask: What coefficients should you use, in a linear combination of  $\vec{b}_1$  and  $\vec{b}_2$ , to get  $\vec{x}$ ?) Then do #15, 16.

Finally, here are three matrices over  $\mathbb{Z}/2\mathbb{Z}$ :

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix},$$

A. Compute  $CE$  as a matrix over  $\mathbb{Z}/2\mathbb{Z}$ . (Remember: You need to do arithmetic modulo 2. In other words, whenever you get an integer greater than 1, you replace it with its remainder after division by 2.)

B. What is the rank of  $E$ ? (You don’t have to compute it in detail. If you can make a guess, that you’re pretty confident about, then I’m satisfied.)

C. What is the nullity of  $C$ ? (Again, a detailed computation is not needed.)

D. Here’s a conceptual question: What do parts A, B, C of this problem have to do with each other?