Your friend Juana sends you the bit string `0111110`. You know that she uses the $(7, 4)$ Hamming code discussed in class. Being her friend, you of course want to know what her message was. I mean, let's not ghost Juana here.

A.A. Assume that the communications line is quite good, meaning that seven-bit messages contain at most one error. Then what do you do?

A.B. Assume that the communications line is pretty good, in that seven-bit messages contain at most two errors. Then what do you do?

B. Working with the $(7, 4)$ Hamming code, give an example where three errors occur in transmission but the error code is `000`. This example shows that we have no hope of detecting more than two errors.

C. The $(7, 4)$ Hamming code was expressed using multiplication-mod-2 involving three matrices, which we called the encoder E, the decoder D, and the checker C. Can the $(5, 4)$ parity bit system (which encodes four bits into five, just as in class) be expressed using multiplication-mod-2 of encoder, decoder, and checker matrices too? If so, then what are these matrices, explicitly, for the parity-bit system?

For Problem D, let

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

be arbitrary 2×2 matrices. "Arbitrary" means that they could be any 2×2 matrices. In other words, you don't know anything special about the numbers A_{ij} or B_{ij}. For example, you don't get to assume that B is diagonal.

D. Prove that $\det(AB) = (\det A)(\det B)$, by explicitly computing out both sides of that equation and comparing the results.

Epilogue: This exercise requires some algebra. It is much easier to see why $\det(AB) = (\det A)(\det B)$ geometrically. We'll do that in class soon. This problem is a great example of why you want to understand linear algebra from multiple directions. Sometimes one direction is easier than another.