First, please do Section 5.1 #5, 6, 15. That last question is asking you: How many linearly independent vectors can you find, that are eigenvectors with eigenvalue 3? (And write down those vectors.)

Second, in class we showed that

$$A = \begin{bmatrix} 5/2 & 1/2 \\ 1/2 & 5/2 \end{bmatrix}$$

has eigenvectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

with associated eigenvalues $\lambda_1 = 2$, $\lambda_2 = 3$ respectively. Let S be the matrix whose columns are $\vec{v}_1, \vec{v}_2$ (in that order). Let D be the diagonal matrix with diagonal entries λ_1, λ_2 (in that order).

Compute SDS^{-1}, simplifying as much as possible and showing all work.

Third, let

$$B = \begin{bmatrix} 1.1 & 0 \\ 0 & 0.9 \end{bmatrix}.$$

Sketch an untransformed object in the plane. Then sketch the object transformed by B, by B^2, by B^3, and by B^4. In a sentence, describe what the object will look like, when it is transformed by B^k for a very large positive integer k. What does that long-term behavior have to do with the eigensystem of B?

Fourth, do Discussion Problem 7B, about long-term pollution in the Great Lakes. If this problem is mystifying to you, then it might help to read the end of Section 5.1, on eigenvectors and difference equations. It might also help to know that this is a five-dimensional version of the third problem of this assignment (the one about the 2×2 matrix B).

If you feel that you want more practice with routine problems such as Section 5.1 #5, 6, then do more problems from the book. You want to be fast at this mechanical task, because we use it to do more complicated tasks later. (But don't hand in this extra practice.)