Please write your name at the top of this cover page and nowhere else.

No notes, books, calculators, computers, etc. are allowed on this exam.

You have 70 minutes.

Show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. **Correct answers without explanatory work rarely earn full credit**.

Perform as much algebraic simplification as you can. Simple correct answers are generally preferred over complicated correct answers. Do simple arithmetic, but don't bother to do complicated arithmetic. Mark your final answer clearly.

Good luck. :)

A. Here are four points in the x-y-plane: $(-2, 0)$, $(0, -1)$, $(1, 1)$, and $(2, -1)$. Use least squares to fit a line $y = mx + b$ to those four points.

You receive the message 1100101 from your Canadian friend Tagak, who uses the $(7,4)$ Hamming code. For your reference, the crucial matrices are

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

B.A. Assume that the communications line is quite good, meaning that seven-bit messages contain at most one error. Then what do you do?

B.B. Alternatively, assume that the communications line is pretty good, in that seven-bit messages contain at most two errors. Then what do you do?

C.A. Suppose that A is an $n \times n$ matrix. Suppose that $\vec{v}$ and $\vec{w}$ are distinct vectors (meaning $\vec{v} \neq \vec{w}$) such that $A\vec{v} = A\vec{w}$. Show that $A\vec{x} = \vec{0}$ has a non-trivial solution $\vec{x}$.

C.B. Compute the determinant of $B = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ 2 & 5 & 2 \end{bmatrix}$.

C.C. For the matrix B above, do there exist distinct vectors $\vec{v} \neq \vec{w}$ such that $B\vec{v} = B\vec{w}$?

D. Is the matrix A below diagonalizable? If not, then explain why; if so, then diagonalize it (meaning, explicitly show an invertible P and a diagonal D such that $A = PDP^{-1}$).

$$A = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}.$$

E. Find a basis for the null space of this matrix:

$$A = \begin{bmatrix} 0 & 2 & -4 \\ 0 & -3 & 6 \\ 0 & 1 & -2 \end{bmatrix}.$$