Before Exam A, I sent e-mail to the class that outlined some kinds of questions that might appear on that exam. The exam ended up looking very much like that outline. However, some students asked for a more detailed/precise/specific set of practice questions for Exam B.

So here are some study questions for Exam B. They do not constitute a promise or contract about what the exam will be like. I have not carefully balanced them for content and difficulty. So they are not a perfect substitute for a practice exam from a previous term (which doesn't exist, because this course is being taught for the first time this term).

Exam B takes place in class on Day 21 (Monday November 4). It covers material up to and including Day 19 and its assigned homework. It is focused on the material that wasn't tested on Exam A. However, some earlier material might appear, just because the course material is inherently cumulative.

Some of the questions here might be slightly too computationally intense for a timed exam, but they're good practice anyway. When basketball players train, they lift weights that are heavier than a basketball.

For Problem A, here are four points in the x-y-plane: $(-1, 0)$, $(0, 1)$, $(1, -1)$, and $(2, 1)$.

A.A. Use least squares to fit a line $y = mx + b$ to those four points.

A.B. Use least squares to fit a parabola $y = ax^2 + bx + c$ to those four points.

B. For each matrix below, find a basis for its column space.

$$A = \begin{bmatrix} 2 & 0 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \\ -1 & 1 & -1 \end{bmatrix}.$$

In Problem C, you receive the message 0000101 from your friend Youssou in Senegal, who uses the $(7, 4)$ Hamming code. For your reference, the crucial matrices are

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

C.A. Assume that the communications line is quite good, meaning that seven-bit messages contain at most one error. Then what do you do?

C.B. Assume that the communications line is pretty good, in that seven-bit messages contain at most two errors. Then what do you do?

D. Suppose that $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ are orthogonal vectors in n dimensions. Explain why it must be true that these vectors are linearly independent.

E. For each matrix below, compute the determinant, and draw a sketch of an object transforming, that illustrates the geometric meaning of the determinant.

$$A = \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ \\ \sin 60^\circ & \cos 60^\circ \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} \cos 30^\circ & 0 & \sin 30^\circ \\ 0 & 1 & 0 \\ -\sin 30^\circ & 0 & \cos 30^\circ \end{bmatrix}.$$

F. !!something about square matrix theorem

G. !!something about eigensystems