For Problem A, let $\Sigma = \{\texttt{a}, \texttt{b}\}$. In class we developed a CFG for the language $A \subseteq \Sigma^*$ consisting of strings w that contain twice as many as as bs.

A.A. Draw the corresponding PDA, according to the proof of Lemma 2.21. To keep your PDA understandable, use our abbreviated notation for pushing an entire string onto the stack. So your PDA will appear to have just three states, as in Figure 2.24.

A.B. How many states does your PDA actually have? Be exact.

B. Prove that the intersection of a context-free language and a regular language is a context-free language. (Need a hint? See the back of this page.)

C. Show that if A is context-free and B is regular, then A/B is context-free. (Need a hint? See the back of this page.)

Here are some hints.

B. What other proofs about intersections have we done in this course?

C. First, mimic your solution to Problem B to make a PDA N that runs a PDA P and a DFA M simultaneously. Then use P and N to construct a PDA for A/B. Various tweaks are required. My solution uses many ϵ-transitions.