A. Exercise 7.4. The given CFG is in Chomsky normal form. So you just need to execute the dynamic programming algorithm described in class and in the textbook.

For the remaining problems, we need some definitions, which we might not have covered yet in class. An undirected graph G is *complete* if there is an edge between every two nodes. In other words, if m is the number of nodes in G, then every one of the

$$\binom{m}{2} = \frac{m(m-1)}{2}$$

possible edges is present in G. In an undirected graph G, a k-*clique* is a set of k nodes that form a complete graph. In other words, every one of the $\binom{k}{2} = k(k-1)/2$ possible edges among them is present. Define

$$\mathit{CLIQUE} = \{\langle G, k\rangle : G \text{ is an undirected graph, } k \geq 1, \text{ and } G \text{ contains a } k\text{-clique}\}.$$

Also, for any $k \geq 1$, let

$$\mathit{CLIQUE}_k = \{\langle G\rangle : G \text{ is an undirected graph that contains a } k\text{-clique}\}.$$

In class, we will soon learn that CLIQUE is NP-complete. Without going into details, this fact implies that if $\mathit{CLIQUE} \in P$, then $P = NP$. The popular belief is that $P \neq NP$ and hence $\mathit{CLIQUE} \notin P$.

B. Show that $\mathit{CLIQUE}_k \in P$ for all k. (For the sake of the next problem, try to pin down your running time fairly precisely. By the way, the $k = 3$ case is Exercise 7.9 in our textbook.)

C. Explain how it's possible that $\mathit{CLIQUE}_k \in P$ for all k, but $\mathit{CLIQUE} \notin P$. In other words, explain why someone might think that $(\forall k\ \mathit{CLIQUE}_k \in P) \Rightarrow \mathit{CLIQUE} \in P$, and why that argument can't be completed.