In class we proved that if $A \leq_p B$ and $B \in P$, then $A \in P$. This first problem addresses the analogous question for NP.

A. If $A \leq_p B$ and $B \in NP$, then must it be true that $A \in NP$? Prove your answer.

For the second problem, don't solve Problem 7.29, but do read it. It generalizes as follows. Define

$$COLOR = \{\langle G, j\rangle : G \text{ is an undirected graph that is colorable with } j \text{ colors}\}.$$

You might wonder what practical value such abstract problems have.

B. Formulate the exam-scheduling problem of Problem 7.31 as a language A. Prove that $A \leq_p COLOR$. (Hint: What must be avoided in an example of A, and what must be avoided in an example of $COLOR$? Those features should correspond to each other. Work backward from that idea.)

The homework to be submitted is just the two problems above. However, it is important that you understand the two polynomial-time mapping reductions done in class today ($3SAT$ to $CLIQUE$ and $3SAT$ to $VERTEX\text{-}COVER$). I recommend that you work concrete examples:

1. Give a satisfiable Boolean formula in 3CNF. Show how it produces an element of $CLIQUE$ and an element of $VERTEX\text{-}COVER$.
2. Give an unsatisfiable Boolean formula in 3CNF. Show how it produces a $\langle G, k\rangle$ that is not in $CLIQUE$ and a $\langle G, \ell\rangle$ that is not in $VERTEX\text{-}COVER$.

Check your answers with a study partner, the prefect, or me. :)