

Notes, book, etc. are not allowed.

Except where otherwise noted, you must justify your answers. Correct answers with no justification may receive little credit. Incorrect or incomplete answers that display insight often receive partial credit.

You may cite material (definitions, theorems, algorithms, etc.) discussed in class, assigned homework, or the assigned textbook sections. If you wish to use other material, then you must develop it first.

It is understood that efficient, concise solutions are usually favored over inefficient or verbose solutions, and hence may earn more points.

If you feel that a problem is ambiguously worded, then ask for clarification. If the problem is still unclear, then explain your interpretation in your solution. Never interpret a problem in a way that renders it trivial.

You have 70 minutes. Good luck. :)

**A.** Let  $A = \{(01)^n : n \geq 0\}$  over  $\Sigma = \{0, 1\}$ . Is  $A$  regular? Prove your answer.

**B.** Let  $A = \{01^i01^j : i \geq j \geq 1\}$  over  $\Sigma = \{0, 1\}$ . Is  $A$  regular? Prove your answer.

**C.A.** In the UK, mobile phone numbers are of the format  $07xxxxxxxx$ , where  $x$  represents any digit (and the  $x$ s need not be identical). Write a textbook regular expression that matches exactly these strings over  $\Sigma = \{0, 1, \dots, 9\}$ .

**C.B.** In the USA, mobile phone numbers are nine-digit strings  $a_0a_1a_2a_3a_4a_5a_6a_7a_8$  subject to three constraints:  $a_0$  cannot be 0 or 1,  $a_3$  cannot be 0 or 1, and  $a_4a_5$  cannot be 11. Write a textbook regular expression that matches exactly these strings over  $\Sigma$ .

**C.C.** Let  $A \subseteq \Sigma^*$  be the set of strings that are neither UK nor USA mobile phone numbers. Does there exist a regular expression to match  $A$ ? Prove your answer.

**D.** On these TRUE-FALSE questions there are four valid answers. If the correct answer is TRUE, then TRUE earns 3 points, TRUISH earns 2 points, FALSISH earns 1 point, and FALSE earns 0 points. If the correct answer is FALSE, then these point values are of course reversed. Do not write just T or F; write your answer completely and clearly. No explanation is needed.

**D.A.** If  $A$  is a finite language, then  $A$  must also be regular.

**D.B.** If  $A$  is regular and  $B \subseteq A$ , then  $B$  must also be regular.

**D.C.** For all languages  $A$  and  $B$ ,  $(A/B)B \subseteq A$ .

**D.D.** If  $A_0, A_1, A_2, \dots$  are infinitely many regular languages, then  $A_0 \cup A_1 \cup A_2 \cup \dots$  must also be regular.

**D.E.** For any regular expression  $R$ , there exists an NFA  $N$  such that  $L(N) = L(R)$  and  $N$  has three states.