

This exam begins for you when you open (or peek inside) this packet. It ends at 2:20 PM on Friday 2020 February 7. Between those two times, you may work on the exam as much as you like. Although I do not intend the exam to require more than a couple of hours, you should get started early, in case you want to spend more time. The exam is open-book and open-note:

- You may use all of this course’s resources: the Sipser textbook, your class notes, your old homework, and the course web site. If you missed a class and want to get some other student’s notes, then do so before leaving class today. You may not share any resources with any other person while you are taking the exam.
- You may cite material (definitions, theorems, examples, algorithms, etc.) from class, the assigned textbooks readings, and the assigned homework problems. You do not have to redevelop or reprove that material. On the other hand, you may not cite results that we have not studied.
- You may not consult any other books, papers, Internet sites, etc. You may use a computer for viewing the course web site, typing and running Python programs of your own creation, typing up your answers, and e-mailing with me. If you want to use a computer for other purposes, then check with me first.
- You may not discuss the exam in any way — spoken, written, etc. — with anyone but me, until everyone has handed in the exam. During the exam period you will inevitably see your classmates around campus. Refrain from asking even seemingly innocuous questions such as “Have you started the exam yet?” If a statement or question conveys any information about the exam, then it is not allowed. If it conveys no information, then you have no reason to make it.

Feel free to ask clarifying questions in person or over e-mail. You should certainly ask for clarification if you believe that a problem is mis-stated. Check your e-mail occasionally, in case I send out a correction.

Your solutions should be thorough, self-explanatory, neat, concise, and polished. You might want to work first on scratch paper and then recopy your solutions. Alternatively, you might want to type your solutions. Always show enough work and justification so that a typical classmate could understand your solutions. If you cannot solve a problem, then write a brief summary of the approaches you’ve tried. Partial credit is often awarded. Present your solutions in the order assigned, in a single stapled packet.

Good luck. :)

A. In a Python file `jdavisIterative2.py` (but named with your Carleton user name instead of mine), solve Problem G from the Recursion tutorial. That is, write the `iterative2` meta-function and demonstrate that it behaves identically to the `recursive2` meta-function on the examples provided. When you are ready to hand in your solution, send e-mail to me with subject “iterative2”, no text in the body, and the Python file attached. To clarify: You are submitting this one problem electronically rather than on paper. (You should not need any more Python skills than what you’ve already used on the homework. Therefore you may not consult any Python documentation or resources. E-mail me Python questions if you must.)

Here are a few problems about intersection and complementation. As always, explain.

B.A. Let $A = \{a^m b^m c^m : m \geq 0\} \subseteq \{a, b, c\}^*$. Write a context-free grammar for A^c . To aid reader comprehension, you might want to comment some parts of your grammar.

B.B. We have proved in class that the complement of a regular language is regular, by switching the accept and non-accept states in a DFA for the former language. Does the same argument applied to PDAs show that the complement of a context-free language is context-free?

B.C. We have proved in homework that the intersection of a context-free language and a regular language is context-free, using a product construction. Does the same argument applied to PDAs show that the intersection of two context-free languages is context-free?

Let $A = \{0^{m^2} : m \geq 0\} = \{\epsilon, 0, 0000, 000000000, \dots\} \subseteq \{0\}^*$.

C.A. Is A context-free? Prove your answer.

C.B. Design a Turing machine to decide A . Your description should be at detail level 2 as specified in the Day 12 homework. You may use multiple tapes if you like.

Let $A = \{a^m b^m c^n : m \geq 0, n \geq 0, m \neq n\} \subseteq \{a, b, c\}^*$. By the way, A is not context-free. (You do not need to prove so.)

D. Prove that A satisfies the conclusions of the CFL pumping lemma, even though it is not context-free. (The purpose of this problem is to test your understanding of the delicate logic of the pumping lemma. So be rigorous.)

E. How much time have you spent on this exam? (Your answer does not affect your exam score, but please do supply an honest answer.)

This paragraph has no relevance to the rest of the exam. It exists just to occupy some of the remaining space on the page. You are not obligated to read it. Anyway, here are some lyrics to a certain hit pop song from the 1980s: Ee do ba be Ee da ba ba ba Um bo bo Be lap People on streets Ee da de da de People on streets Ee da de da de da de da. And here’s the bass line: Dum dum dum dum de de dum dum. And here’s the piano part: Plink plink.