

In addition to this cover page, there are four problems (A–D.C) spread over five pages.

No notes, books, calculators, computers, etc. are allowed.

Feel free to ask clarifying questions. If a problem is unclear and you cannot obtain clarification, then write your interpretation of the problem, so that I can evaluate your solution relative to your interpretation. You might be penalized, if your interpretation makes the problem much easier than it should be. Certainly you should never interpret a problem in a way that renders it trivial.

Write as if your audience is a typical classmate — not a professor. In doing so, you (hopefully) show enough detail, that I can evaluate whether you understand your arguments.

You may cite material (definitions, algorithms, theorems, etc.) that we have defined or proved in class, in the assigned textbook readings, or in the assigned homework. You do not need to re-define or re-prove any of that material. You may not cite other material without developing it first.

You have 70 minutes. Good luck. :)

A. Let $A = \{0^j 10^k : j \geq k \geq 0\}$. Is A regular? Prove your answer.

B. For any integer $k \geq 0$, let $A_k = \{0^j 10^k : j \geq k\}$. Is A_k regular for all k ? Prove your answer.

C. Draw a DFA or NFA whose language equals the language of this regular expression:

$$((01)^* \cup 0^*)^* 1^* \cup 10^*$$

As part of your research in natural language processing, you are exploring how well CFGs can describe English grammar. So far, you are studying only sentences made up of nouns, adjectives, verbs, and adverbs. Here's your current CFG, with Se as start variable:

$$\begin{aligned} Se &\rightarrow Np Vp \mid Np Vp Np \\ Np &\rightarrow No \mid Adj Np \\ No &\rightarrow \text{mouse} \mid \text{cat} \mid \text{kumquat} \mid \text{dignity} \\ Adj &\rightarrow \text{brown} \mid \text{the} \mid \text{smart} \\ Vp &\rightarrow Ve \mid Adv Ve \\ Ve &\rightarrow \text{eats} \mid \text{covets} \mid \text{questions} \\ Adv &\rightarrow \text{idly} \mid \text{seldom} \mid \text{sarcastically} \end{aligned}$$

D.A. Draw the parse tree for the sentence “the kumquat seldom covets the smart brown cat”. A tree node should contain a variable if and only if it is a branch node.

D.B. According to this CFG, can the sentence “the kumquat seldom covets smart the brown cat” be parsed? Explain.

D.C. Is the language of this CFG regular? Explain thoroughly.