A. No, this language A is not regular, as I now prove. Assume for the sake of contradiction that A is regular. Let p be a pumping length for A. Let $w = 0^p10^p$. Then $w \in A$ and $|w| = 2p+1 \geq p$, so w can be pumped. In the usual way, $w = xyz$, where $|xy| \leq p$. So y is a substring of the first 0^p in w, and $y = 0^k$ for some k such that $1 \leq k \leq p$. The pumping lemma says that

$$xy^0z = xz = 0^{p-k}10^p$$

is another string in A. However, this string is not in A, because $p - k < p$. This contradiction implies that A is not regular.

B. Yes, A_k is regular for all k. We can demonstrate so by exhibiting a regular expression for each A_k:

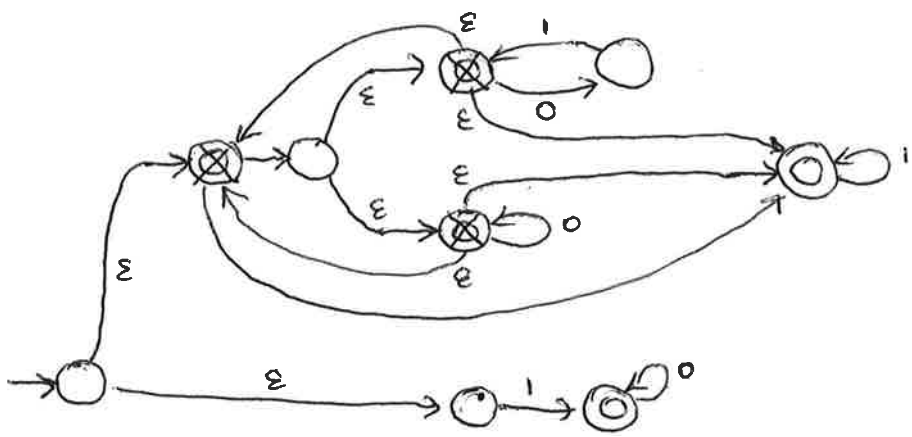
$$\begin{aligned} A_0 &= L(0^*1), \\ A_1 &= L(00^*10), \\ A_2 &= L(000^*100), \end{aligned}$$

and in general

$$A_k = L(0^k0^*10^k).$$

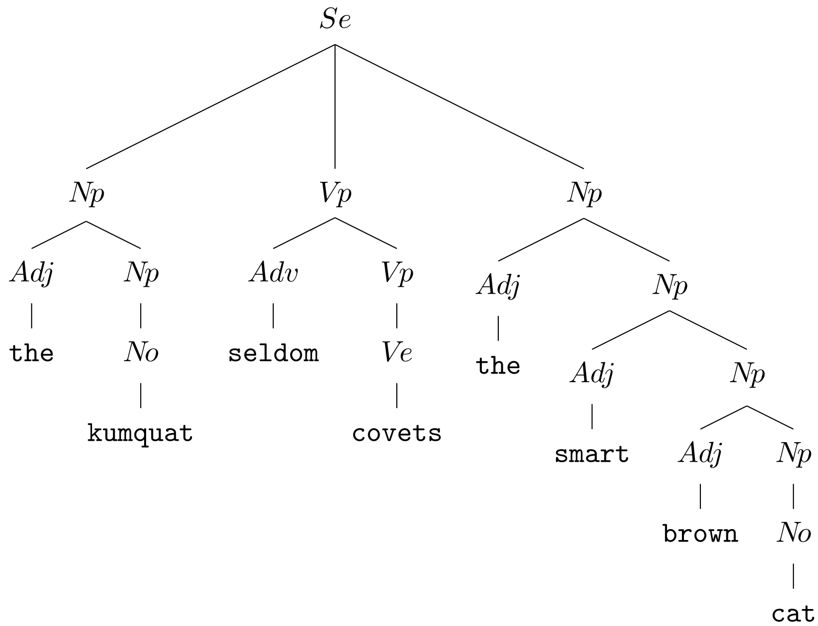
[Alternatively, if you draw DFAs for A_0, A_1, and A_2, then you can quickly convince yourself that you can draw a DFA for any A_k.]

C. Here is my NFA:



States, that look like accept states Xed out, are non-accept states. [I constructed this NFA following our usual algorithm for converting a regular expression to an NFA, but with a couple of shortcuts.]

D.A. Here is the parse tree:



[The CFG given above has a small error. The *Vp* rule should be $Vp \to Ve \mid Adv\ Vp$. The parse tree above reflects that corrected rule. If you use the CFG with the incorrect form of the rule, then the sentence still parses, with a slightly simpler parse tree.]

D.B. Yes, the new sentence parses. We get the same parse tree as in Problem D.A, except that the `the` and `smart` nodes are switched. [The CFG is not sophisticated enough to capture the fact that this new sentence is not actually grammatical in English.]

D.C. Yes, the language of this CFG happens to be regular. The *Np* variable of the CFG generates the language of the regular expression

$$(\texttt{brown} \cup \texttt{the} \cup \texttt{smart})^*(\texttt{mouse} \cup \texttt{cat} \cup \texttt{kumquat} \cup \texttt{dignity}).$$

Similarly, the *Vp* variable of the CFG generates the language of the regular expression

$$(\texttt{idly} \cup \texttt{seldom} \cup \texttt{sarcastically})^*(\texttt{eats} \cup \texttt{covets} \cup \texttt{questions}).$$

[This regular expression is based on the corrected form of the CFG. The incorrect CFG, that actually appears on the exam, yields a simpler regular expression.] Therefore the language of the CFG is the language of the regular expression

$$NV \cup NVN,$$

where N and V are the regular expressions just described, respectively.