This exam must be completed within a single three-hour block, of your choosing, before class on Monday. It is recommended, but not required, that you take the exam between 6:00 AM and 10:00 PM, when I am most likely to check e-mail.

The exam is open-note: You may use your class notes, your old homework, your textbook, and the course web site. You are not allowed to share any of these materials with other students. You are not allowed to discuss the exam with anyone but me until Monday at the start of class. You are not allowed to consult other Internet sites, artificial intelligences, books, etc.

I will try to check my e-mail frequently during the exam period. Feel free to ask clarifying questions. If you cannot obtain clarification on a problem, then explain your interpretation of the problem in your solution, so that I can judge your solution relative to your interpretation. You might lose points, if your interpretation makes a problem drastically easier than it should be. Certainly you should never interpret a problem in a way that renders it trivial.

Your solutions should be thorough, self-explanatory, neat, concise, and polished. Always show enough work and justification so that a typical classmate could understand your solutions. Correct answers without supporting work rarely earn full credit. You may cite material (definitions, theorems, examples, algorithms, etc.) from class, homework, assigned textbook readings, etc. You do not have to redevelop or reprove that material. If you wish to use material that we haven't studied, then you have to develop it.

If you cannot solve a problem, then write a brief summary of what you know that is relevant, and the approaches that you've tried. Partial credit is often awarded.

Good luck. :)

For the first problem, let $A = \{0^j 1^k 0^j : j \geq k \geq 0\}$.

A. Prove that A is not context-free.

In a directed graph G, the *out-degree* of a vertex v is the number of edges that start at v and go to other vertices. Let $A = \{\langle G \rangle : \text{ every vertex of } G \text{ has out-degree } 2\}$. Let's agree, for the sake of this problem, that edges from a vertex to itself are not permitted. Let's agree that $\langle G \rangle$ is some version of the adjacency matrix of G.

B.A. Draw an example G and write its corresponding $\langle G \rangle$, to clarify exactly which encoding system you're using.

B.B. Describe a decider for A. Your description should be at Moderate Level as described on the Day 12 Homework — roughly speaking, a description of how the tape head traverses and marks the tape to accomplish its work, but not a complete drawing of the state diagram.

In this next problem, we're going to show that this language is undecidable:

$$A = \{\langle M \rangle : M \text{ is a TM, and there exists a string } x \text{ such that } M \text{ halts on input } x\}.$$

C.A. Is the undecidability of A an immediate consequence of Rice's theorem?

C.B. Prove that A is undecidable. Regardless of your answer to Problem C.A, you are not allowed to use Rice's theorem in this proof. (Hint: The relationship between $EMPTY_{TM}$ and ACC_{TM} is similar to the relationship between A and $\overline{HALT_{TM}}$.)

One night in the Cave you find a mangled paper with the following computability theory argument on it. Two parts of the argument are illegible: the definition of A, and the reasoning for why $\overline{ACC_{TM}}$ is the language of R. Here is a cleaned-up version:

Let $A = \ldots$. Assume for the sake of contradiction that A is recognizable. Let H be a recognizer for A. I now describe a recognizer R for $\overline{ACC_{TM}}$. This Turing machine R, on input $\langle M, w \rangle$, does these steps:

1. Build a Turing machine N that, on input x, does these steps:
   - (a) Simulate M on input w, but only for the first $|x|$ steps of M's computation.
   - (b) If by this point M has accepted w, then enter into an infinite loop (hang).
   - (c) Otherwise, accept.
2. Run H on $\langle N \rangle$ and output whatever H outputs.

To verify that $\overline{ACC_{TM}}$ is the language of R, ... But we already know that $\overline{ACC_{TM}}$ is not recognizable. This contradiction implies that A is not recognizable either.

D. In the preceding argument, what is A?