In addition to this cover page, this exam should have five pages of problems labeled A–E.

No notes, books, calculators, computers, etc. are allowed, except for one two-sided, one-sheet crib sheet as we have discussed.

Feel free to ask clarifying questions. If a problem is unclear and you cannot obtain clarification, then write your interpretation of the problem, so that I can evaluate your solution relative to your interpretation. You might be penalized, if your interpretation makes the problem much easier than it should be. Certainly you should never interpret a problem in a way that renders it trivial.

You may cite material (definitions, algorithms, theorems, etc.) that we have defined or proved in class, in the assigned textbook readings, or in the assigned homework. You do not need to re-define or re-prove any of that material. You may not cite other material without developing it first.

Show your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

Write as if your audience is a typical classmate — not a professor. In doing so, you (hopefully) show enough detail, that I can evaluate whether you understand your arguments.

You have 150 minutes to complete this exam. Good luck. :)

A.A. Deciding whether a string of parentheses is a valid nest is one of our classic PDA problems. How much stack space did our PDA solution use?

A.B. Describe a Turing machine for deciding this problem using only logarithmically more space than is needed to store the input. Not much detail is expected, but you certainly need enough detail to explain the space usage.

B.A. Assuming that your reader knows what a *cut* of an undirected graph is, define $MAX\text{–}CUT$.

B.B. Sketch a proof that $MAX\text{–}CUT \in NP$.

I now argue that $3SAT \leq_p \neq SAT$. To begin the argument, I describe a Turing machine F. On input $\langle \phi \rangle$, where ϕ is a Boolean formula ϕ in 3CNF, F does these steps:

1. Let k be the number of clauses in ϕ. Allocate space for a new 3CNF formula ψ that has $2k$ clauses, all of the variables of ϕ, and $k+1$ new variables, which are called $w_1, w_2, \ldots, w_k, b$.
2. For $i = 1, \ldots, k$, let $(x \vee y \vee z)$ be the ith clause in ϕ. (Here, each of x, y, and z can be any variable or its negation.) Add two clauses $(x \vee y \vee w_i) \wedge (\overline{w_i} \vee z \vee b)$ to ψ.
3. Output $\langle \psi \rangle$, where ψ is the 3CNF formula of $2k$ clauses that was just produced.

C. To finish the argument above, what things need to be checked? Your responses should be specific to the languages above, rather than general and abstract. (I am not asking you to check these things. I am just testing whether you know everything that needs checking.)

In each part below, give a brief explanation of why the given relationship is true — maybe just a single sentence giving the key idea of the proof, as if you're trying to jog the memory of a classmate.

D.A. $P \subseteq NP$

D.B. $NPSPACE \subseteq PSPACE$

D.C. $P \subseteq PSPACE$

D.D. $PSPACE \subseteq EXPTIME$

E. It is illegal and non-sensical for a TM or NTM to alter a tape cell that is far away from the tape head. In the proof of the Cook-Levin theorem, what prevents the formula ϕ from being satisfied, if the tableau contains such an illegal transition?