Because this is our first homework assignment, you might benefit from some clarification. This homework is assigned on Day 01 (Wednesday). You are expected to attempt it immediately, before Day 02 (Friday). For then you better understand Day 02 of class, and you have time to discuss the homework with me and others before it's due. Hand in your solutions on paper at the start of class on Day 03 (Monday), with "Day 01" written at the top. If your solutions use multiple pages of paper, then please staple them. Is "Day 1" just as good as "Day 01"? Yes. Is a paper clip just as good as a staple? No.

There are four exercises labeled A–D. Insofar as they are difficult, the difficulty mostly lies in figuring out what they are asking. After that, the solutions are often quite short.

On page 11, Munkres draws a diagram to illustrate that intersection distributes over union: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

A. Draw a similar diagram to verify the other distributive law, which says that union distributes over intersection: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Let $X$ and $Y$ be non-empty sets, and consider the *projection* $p : X \times Y \to X$ defined by $p(x, y) = x$. (There is another projection $q : X \times Y \to Y$, but let's just do $p$.)

B. Under exactly which conditions is $p$ surjective? Under exactly which conditions is $p$ injective? Prove your answers.

C. Do Section 3 Exercise 3 (a flawed argument about equivalence relations).

Finally, we already know (from Math 236 or page 23 of Munkres) that equivalence relations and partitions are essentially the same thing. This last exercise shows that equivalence relations and surjections are essentially the same thing. Let $X$ be a set, $\sim$ an equivalence relation on $X$, and $X/\sim$ the set of equivalence classes. Let's denote the equivalence class of $x$ under $\sim$ as $[x]_\sim$. The function $f_\sim : X \to X/\sim$ defined by $f_\sim(x) = [x]_\sim$ is a surjection. So, from an equivalence relation $\sim$, we get a surjection $f_\sim$ in a natural way. For the converse, let $Y$ be a set and $f : X \to Y$ a surjection.

D.A. Use $f$ to define a natural equivalence relation $\sim_f$ on $X$, such that the rest of this problem is doable.

D.B. Prove that $\sim_{f_\sim} = \sim$.

D.C. Find a bijection $g : X/\sim_f \to Y$ such that $g \circ f_{\sim_f} = f$.

Therefore the operations of viewing an equivalence relation as a surjection, and viewing a surjection as an equivalence relation, are inverses of each other (up to $g$).