

A. Section 16 Exercise 1

B. In  $\mathbb{R}$ , find a homeomorphism between  $(0, 1)$  and any other finite open interval  $(a, b)$ . Also find a homeomorphism between  $(0, 1)$  and  $\mathbb{R} = (-\infty, \infty)$ . (You may use the fact, which we have not proved yet, that continuity in  $\mathbb{R}$  and its subspaces matches continuity as defined in calculus.)

C. Let  $Y = \{1, 1/2, 1/3, 1/4, 1/5, \dots\}$  and let  $Z = Y \cup \{0\}$ , both with their subspace topologies from  $\mathbb{R}$ . Is  $Y$  discrete (meaning, a space with the discrete topology)? Is  $Z$  discrete?

D. What happens if you carry out the construction of the quotient topology on  $Y$ , for a function  $f : X_T \rightarrow Y$  that is not surjective?

E. Consider the surjection  $f : \mathbb{R}_{\text{std}} \rightarrow [0, \infty)$  given by  $f(x) = x^2$ . Endow  $[0, \infty)$  with the quotient topology induced by  $f$ . How does it compare to the subspace topology on  $[0, \infty) \subseteq \mathbb{R}_{\text{std}}$ ?

(Also, I invite you to consider the surjection  $f : \mathbb{R}_{\text{std}} \rightarrow \mathbb{R}$  given by an odd-degree polynomial. What is the quotient topology on  $\mathbb{R}$ , if  $f(x) = x^3$ ? If  $f(x) = x^3 + x$ ? If  $f(x) = x^3 - x$ ? But you are not asked to hand in answers to these questions.)