(When I posted this homework earlier today, the problems were not finalized, because I didn't know whether we'd cover enough material in class. But we did — barely. So these are the finalized problems.)

A. Section 16 Exercise 4. (A side effect of this exercise is the definition of *open map*, which is fairly important.)

In class on Day 03, we used the surjection $f : \mathbb{R} \to [0, 1)$ defined by $f(x) = x - \lfloor x \rfloor$ to endow $[0, 1)$ with a non-standard topology T (the quotient topology induced by f). I argued intuitively that $[0, 1)_T$ is a circle. Let's make that more rigorous. I claim that $g : [0, 1)_T \to \mathbb{S}^1 \subseteq \mathbb{R}^2$ defined by $f(x) = (\cos 2\pi x, \sin 2\pi x)$ is a homeomorphism. I take it for granted that g is bijective onto $\mathbb{S}^1$.

B. Prove that g is an open map. (This task is not especially difficult, but your proof should explicitly address the fact that there are two kinds of open sets in $[0, 1)_T$: the ones that contain 0, and the ones that don't.)

Epilogue: A similar proof establishes that g is continuous and hence a homeomorphism. I am not asking you to prove that g is continuous. Also, in class on Day 04, we showed that $g : [0, 1)_{\text{std}} \to \mathbb{S}^1$ is not a homeomorphism. We did not show, but we will eventually show, that no homeomorphism exists between $[0, 1)_{\text{std}}$ and $\mathbb{S}^1$. Then we can conclude that $[0, 1)_T$ and $[0, 1)_{\text{std}}$ are not homeomorphic.

C. Section 13 Exercise 8a. (You may use the fact that between any two real numbers there is a rational number. You may use Lemma 13.2, although we have not discussed it in class. It is similar to Lemma 13.3, which we will prove. We don't have time for every theorem. I assign this homework problem partially to get you to wrestle with Lemma 13.2 a bit.)

Epilogue: Section 16 Exercise 6 is similar to Section 13 8a. Do it, if you want more practice, but I'm not asking you to hand it in.

D. Section 13 Exercise 5. Do the basis part only; skip the sub-basis part (although it's good practice of course, once we study what a sub-basis is).