

A. Section 17 Exercise 11, which is about the product of two Hausdorff spaces.

B. Section 17 Exercise 12, which is about subspaces of Hausdorff spaces.

C. Fix integers $m \geq n \geq 0$. Let X be a subspace of \mathbb{R}^m such that every $x \in X$ has a neighborhood that is homeomorphic to an open set in \mathbb{R}^n . Prove that X is an n -manifold. (Hint: You might want to cite Lemma 16.1.)

Epilogue: My understanding of history is that manifolds were originally viewed as subspaces of \mathbb{R}^m . Then mathematicians started preferring a more abstract, intrinsic treatment that didn't assume an embedding into \mathbb{R}^m . So the question naturally arose: Even if you don't assume an embedding, does an embedding exist anyway? This exercise shows that the existence of an embedding requires the two technical axioms of n -manifolds. We might or might not have time in this course, to prove the converse: The two technical axioms imply the existence of an embedding. One version of this result is in Section 36.

By the way, Section 17 Exercise 13 is a famous classic, which you might want to try. But I'm not asking you to hand it in.