A. By adapting stereographic projection as described in class, find a homeomorphism between $\mathbb{S}^1 - \{(-1, 0)\}$ and $\mathbb{R}$, and find a homeomorphism between $\mathbb{S}^1 - \{(1, 0)\}$ and $\mathbb{R}$. (This is a short exercise.)

In class, I mentioned that paracompact Hausdorff spaces enjoy a feature called partition of unity, which can be used to prove many theorems. Let's not delve into paracompactness, but let's do one exercise on partition of unity, which is defined in Section 36 of Munkres. For this exercise, recall that the circle $\mathbb{S}^1 \subseteq \mathbb{R}^2$ is a compact 1-manifold.

B. Following the proof of Theorem 36.2, construct an embedding of $\mathbb{S}^1$ into some $\mathbb{R}^N$. In particular, what value of N do you get? (This is a medium-length exercise. You should be able to explicitly specify sets U_i, functions ϕ_i and g_i, etc. Your solution should include graphs/pictures of the ϕ_i.)

I'd like to give you an exercise about homotopy too, but I'm not sure how far we'll get in class today, so let's leave this homework at two exercises.