In class we used a "pasting lemma" about functions on closed subsets. It is exactly Lemma 18.3 in Munkres. He proves it using the closed-set definition of continuity, which is Theorem 18.1 (1) vs. (3).

A. Prove the same pasting lemma, but without citing (or reproving) the closed-set definition of continuity. That is, your argument should be of the form "let $U \subseteq Y$ be open, blah blah blah, so $h^{-1}(U) \subseteq X$ is open". (This is a medium-length exercise that uses your knowledge of set theory heavily.)

My main goal for the preceding exercise is to get you to engage with the pasting lemma (beyond just reading it). A secondary goal is for you to formulate an opinion of whether Munkres's approach is better or worse than this approach.

On page 327, Munkres implicitly states this lemma:

Lemma: If $f_0, f_1 : I \to X_T$ are path-homotopic and $k : X_T \to Y_S$ is any map, then $k \circ f_0$ and $k \circ f_1$ are paths in Y_S and $k \circ F$ is a path homotopy between them. Moreover, if $f, g : I \to X_T$ are paths such that $f(1) = g(0)$, then $k \circ (f * g) = (k \circ f) * (k \circ g)$.

B. Prove the lemma above. (This is a short exercise. Insofar as it is difficult, the difficulty lies in figuring out what it is asking.)

C. Section 51 Exercise 3ab. (This is a medium-length exercise. Parts c and d are valuable too, but I am not asking you to hand them in.)