This is the last homework assignment that you will hand in. I hope that it's a bit fun.

Recall from class that the *Euler characteristic* $\chi = V - E + F$ can be defined for any "well-behaved" division of a compact surface into V vertices, E edges, and F faces. The notion of well-behavedness is made precise in Munkres's definition of a triangulation in Section 78. The only difference here is that we allow faces with more than three sides. Such faces can be subdivided into triangles to yield a triangulation in the sense of Munkres. So the difference is superficial.

Topologically, a *Platonic solid* is a sphere divided into vertices, edges, and faces in a well-behaved way, with two additional stipulations. First, there is a positive integer $D \geq 3$ such that exactly D edges meet at each vertex. Second, there is a positive integer $S \geq 3$ such that each face has exactly S sides. (If the word "solid" confuses you, what we're calling a Platonic solid is actually the boundary surface of a Platonic solid.)

A. Enumerate all possible combinations of D and S. (This is a medium-length or possibly long exercise. If you get stuck, then consult the three-step solution sketch on the back of this sheet.)

A classic soccer ball (such as the Adidas Telstar) is made from H hexagons and P pentagons sewn together along their sides, such that exactly three faces meet at each vertex. Topologically, it's a well-behaved division of a sphere.

B. Which values of P are possible? (This is a medium-length or possibly long exercise. If you get stuck, then consult the two-step solution sketch on the back of this sheet.)

Let X and Y be compact surfaces. Then one can prove that the disjoint union $X \sqcup Y$ (defined on Exam B) and the connected sum $X \# Y$ are also compact surfaces.

C. How does $\chi(X \sqcup Y)$ relate to $\chi(X)$ and $\chi(Y)$? How does $\chi(X \# Y)$ relate to $\chi(X)$ and $\chi(Y)$? Give convincing explanations but not necessarily rigorous proofs. (This exercise is short or medium-length.)

Here are three stepping stones to solving problem A.

A.A. Prove that $VD = 2E = FS$.

A.B. Prove that $\frac{1}{D} + \frac{1}{S} > \frac{1}{2}$.

A.C. Based on problem A.B, enumerate all possible combinations of D and S.

Here are two stepping stones to solving problem B.

B.A. Prove that P must be a multiple of 2 and a multiple of 3.

B.B. Prove that $P = 12$ is the only possible value of P.