1. Here are six concepts: topology, basis, subbasis, metric, inner product, norm. Describe precisely how they relate to each other; which induces which, and how?

2. Let X be any Hausdorff space. Prove that any one-point subset $\{x\}$ of X is closed.

3. Suppose Y is a subspace of X and A a subset of Y. Answer ONE of the following. Mark a giant X through the other one. There is no extra credit for answering both.

A. Is the closure of A in Y equal to the closure of A in X? Prove or give a counterexample.

B. Is the interior of A in Y equal to the interior of A in X? Prove or give a counterexample.

4. Let $X = [-1,1] \times (-1,1) \subseteq \mathbb{R}^2$ in the subspace topology. Let $Y = [-1,1) \times (-1,1)$ as a subset of $\mathbb{R}^2$. Define $f : X \to Y$ by

$$f(x,y) = \begin{cases} (x,y) & \text{if } x \neq 1, \\ (-1,-y) & \text{if } x = 1. \end{cases}$$

Endow Y with the quotient topology from f. In words and/or pictures, describe Y as a space. Describe its open sets. Is it a manifold? (Your answers to this problem need not be rigorous, but try to explain as well as you can.)

5. Let Y be any topological space. Let F be the set of all continuous functions $f : \mathbb{R} \to Y$. For any closed interval $C \subseteq \mathbb{R}$ and open $U \subseteq Y$, let

$$S(C, U) = \{f : f(C) \subseteq U\} \subseteq F.$$

Let T be the topology on F generated by all of these subsets $S(C, U) \subseteq F$. Finally, define a function $e : \mathbb{R} \times F \to Y$ by $e(x, f) = f(x)$. Prove that e is continuous.