No notes, books, calculators, computers, etc. are allowed.

If a problem is unclear and you cannot obtain clarification, then write your interpretation of the problem, so that I can evaluate your solution relative to your interpretation. You might be penalized, if your interpretation makes the problem much easier than it should be. Certainly you should never interpret a problem in a way that renders it trivial.

Write as if your audience is a typical classmate — not a professor. In doing so, you (hopefully) show enough detail, that I can evaluate whether you understand your arguments.

You may cite material (definitions, theorems, etc.) that we have defined or proved in class, in the assigned textbook readings, or in the assigned homework. You do not need to re-define or re-prove any of that material. You may not cite other material without developing it first.

Some problems ask for proofs. On a short exam such as this, you might not have time to make every proof rigorous. Try to hit all of the important concepts in your proof, even if it means leaving logical gaps. Mark each logical gap, so that I know that you know that it is a gap. For example, you might write "Claim: . . . ", finish the proof using the claim, and circle back to prove the claim only if you have time.

Pictures often help both you and your reader.

You have 60 minutes. Good luck. :)

Let $f : X_T \to Y_S$ be continuous. Let $g : Z_R \to W_Q$ be continuous. Define $h : X \times Z \to Y \times W$ by $h(x, z) = (f(x), g(z))$.

A. Prove that h is continuous.

B. Let X_T be a space with the discrete topology. Prove that X_T is metrizable.

One day, you are trying to prove that the continuous image of a Hausdorff space is Hausdorff...

"Theorem: Let X_T be Hausdorff and $f : X_T \to Y_S$ continuous. Then $f(X)$ is Hausdorff.

"Proof: Let $x \neq y \in f(X)$. Let $z \in f^{-1}(x)$ and $w \in f^{-1}(y)$. Then $z \neq w$. Because X_T is Hausdorff, there exist disjoint open sets $U, V \subseteq X_T$ such that $z \in U$ and $w \in V$. Then $f(U)$ and $f(V)$ are disjoint and open in $f(X)$, and $x \in f(U)$ and $y \in f(V)$. Because $x \neq y$ were arbitrary in $f(X)$, we conclude that $f(X)$ is Hausdorff."

Upon reading this text of yours, your friend responds: "Sorry, but I don't understand some steps of your argument. In fact, I doubt that the claimed theorem is true. Instead of assuming that X_T is Hausdorff and proving that $f(X)$ is Hausdorff, you should try going the other way. Maybe that will work?"

C. Based on your friend's suggestion, state the correct version of the theorem. (It is recommended that you also write a proof, to check that your proposed theorem is actually true. But the proof is not worth any points in itself.)

On another day, your friend is trying to prove a theorem about connectedness...

"Theorem: Let Y_1 and Y_2 be connected subspaces of X_T. Then $Y_1 \cup Y_2$ is connected.

"Proof: Suppose, for the sake of contradiction, that there exists a separation of $Y_1 \cup Y_2$. That is, $Y_1 \cup Y_2$ can be partitioned into non-empty open sets U and V. Let $U_1 = Y_1 \cap U$, $V_1 = Y_1 \cap V$, $U_2 = Y_2 \cap U$, and $V_2 = Y_2 \cap V$. Then U_1 and V_1 constitute a separation of Y_1, and U_2 and V_2 constitute a separation of Y_2. But Y_1 and Y_2 are connected. This contradiction implies that $Y_1 \cup Y_2$ is also connected."

D.A. Is your friend's proposed proof correct? If not, then specifically where is it flawed?

D.B. Is the proposed theorem itself correct? Discuss.

D.C. If the theorem is not correct, then how might you add hypotheses, to make it correct? (This is an open-ended question. There may be multiple correct answers. In general, simple answers are preferable. Some explanation is expected, but not a proof.)